Key Exam Topics in This Lesson

Basic Statistics
- Basic Sample Population Statistics
- Skew and Kurtosis
- Normal Distribution (a.k.a. Elliptical or Gaussian)
- Typical Risk Distribution
- Types of Correlation Measures
- BA II Plus Calculator Practice!
Basic Sample Population Statistics

For a univariate sample of $T$ observations:

Sample Mean $= \bar{X} = \frac{1}{T} \sum_{t=1}^{T} X_t$

Median $= 50$th percentile

Mode $= \text{the most common observation}$

Sample Variance $= s^2 = \frac{1}{T-1} \sum_{t=1}^{T} (X_t - \bar{X})^2$

Range $= \max X_t - \min X_t$

For a 2-variable sample (correlation and covariance):

$$r_{X,Y} = \frac{s_{X,Y}}{s_X s_Y}$$
$$s_{X,Y} = \frac{1}{T-1} \sum_{t=1}^{T} (X_t - \bar{X}) (Y_t - \bar{Y})$$

Skew and Kurtosis

**Skew** $= \text{third central moment, normalized}$

$$\omega = \frac{1}{T} \left( \frac{\sum_{t=1}^{T} (X_t - \mu)^3}{\sigma^3} \right) \quad \text{(Population)}$$

$$w = \left( \frac{T}{(T-1)(T-2)} \right) \left( \frac{\sum_{t=1}^{T} (X_t - \bar{X})^3}{s^3} \right) \quad \text{(Sample)}$$

**Kurtosis** $= \text{fourth central moment, normalized against Gaussian distribution}$

$$\kappa = \frac{1}{T} \sum_{t=1}^{T} \frac{(X_t - \mu)^4}{\sigma^4} - 3 \quad \text{(Population)}$$

$$k = \left( \frac{T(T+1)}{(T-1)(T-2)(T-3)} \right) \left( \frac{\sum_{t=1}^{T} (X_t - \bar{X})^4}{s^4} \right) - \frac{3(T-1)^2}{(T-2)(T-3)} \quad \text{(Sample)}$$
Normal Distribution (a.k.a. Elliptical or Gaussian)

- Mean = mode = median
- Skew = 0
- Kurtosis = 3 ("mesokurtic")

Typical Risk Distribution

- Skew < 0: long left tail ⇒ many more worse-than-expected results
- Kurtosis > 3 ("leptokurtic"): fatter tails ⇒ higher probability of extreme results
Types of Correlation Measures

1. **Pearson’s rho** – a.k.a. linear correlation

\[
\rho = \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y} \quad r_{X,Y} = \frac{S_{X,Y}}{s_X s_Y}
\]

2. **Spearman correlation** = Pearson linear correlation of the observations’ ranks

\[
\rho_s = \rho [\text{Ranks of } X, \text{Ranks of } Y]
\]

3. **Kendall’s tau** – measures the observations’ tendency to move together

\[
\tau = \frac{\text{Concordant Pairs} - \text{Discordant Pairs}}{\text{Total Possible Pairs}}
\]

4. **Tail correlation** – correlation of tail values only

**BA II Plus Calculator Practice!**

Suppose we have 5 observations of \( X \) and \( Y \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>( X_t )</th>
<th>( Y_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>30</td>
</tr>
</tbody>
</table>

Calculate the following statistics using your BA II Plus calculator:

1. Sample mean of \( X \) and \( Y \)
2. Sample standard deviation of \( X \) and \( Y \)
3. Covariance of \( X \) and \( Y \)
4. Linear correlation of \( X \) and \( Y \)
No peeking!

BA II Plus Calculator Practice!

See video for a walk through of the BA II Plus’s statistical functions!

<table>
<thead>
<tr>
<th>$t$</th>
<th>$X_t$</th>
<th>$Y_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>30</td>
</tr>
</tbody>
</table>

Sample mean $\bar{X} = 40.00$  $\bar{Y} = 20.00$

Sample standard deviation $s_X = 33.91$  $s_Y = 7.91$

Pearson’s correlation coefficient $r_{X,Y} = \frac{\sum(X_t - \bar{X})(Y_t - \bar{Y})}{s_X s_Y} = \frac{150}{33.91 \times 7.91} = 55.92\% = \text{Pearson’s rho}$