The Infinite Actuary's

Drill Problems for the

LRM Exam

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Introduction

- This document has a number of practice problems to test your LRM knowledge
- There are many different kinds of problems:
  - Verbal vs quantitative
  - Easy vs challenge problems
  - Questions you have seen in the course videos, vs entirely new questions for new practice
- If the challenge problems are overly difficult for you, do not sweat it. This is just extra practice to give you some additional problems under your belt, but don't get too hung up on any of the challenge problems
- You will notice that some readings have a heavier focus / larger number of drill problems.
  - We wanted to add drill problems to help solidify challenging topics (e.g. xVA Challenge, Section 4 / Duration, portfolio variance calculations, etc.).
  - Additionally, LRM-126 has end-of-chapter questions with solutions, so we wanted to include those for additional practice.
- Keep in mind there are also 3 TIA LRM practice exams. These represent what we think is an even mix of LRM topics at exam-level difficulty.

Good luck and enjoy! ☺

1. (LRM-100)

The following approximation to risk capital is made for three different businesses:

$$\text{Risk Capital} \approx .4A_0\sigma\sqrt{T}$$

You are given the following correlation matrix and volatilities:

$$\Sigma = \begin{bmatrix} 1 & .5 & 0 \\ .5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \sigma_1 = 37.5\%, \sigma_2 = 50\%, \sigma_3 = 62.5\%$$

Suppose that each business has $1,000 of gross assets.

Calculate the approximate risk capital of the three combined businesses over a time period of one year.

46. (The xVA Challenge, Jon Gregory)

Assume future values follow a multivariate normal distribution with mean = 0 and equal variance. Recall that the netting factor is given by:

$$\text{Netting Factor} = \frac{\sqrt{n + n(n-1)\overline{\rho}}}{n}$$

(a) State the possible values $[a; b]$ of the netting factor.

(b) What value of $\overline{\rho}$ corresponds to the minimum possible value a?

(c) What value of $\overline{\rho}$ corresponds to the maximum possible value b?

63. (The xVA Challenge, Jon Gregory)

Suppose you are given the following simulation results:

Simulation Path	Value^1	Value^2
1	50	-45
2	-20	20
3	30	15
4	10	-5
5	15	-10

Value^k denotes the value of trade k.

Calculate the following:

(i) Expected Exposure (EE) for:
  (a) Trade 1 Only
  (b) Trade 2 Only
  (c) Trade 1 + 2 (No Netting)
  (d) Trade 1 + 2 (With Netting)

(ii) Negative Expected Exposure (NEE) for:
  (a) Trade 1 Only
  (b) Trade 2 Only
  (c) Trade 1 + 2 (No Netting)
  (d) Trade 1 + 2 (With Netting)

82. (LRM-126)

A DI has the following assets in its portfolio: \$20 million in cash reserves with the Fed, \$20 million in T-Bills, \$50 million in mortgage loans, and \$10 million in fixed assets.

If the assets need to be liquidated at short notice, the DI will receive only 99 percent of the fair market value of the T-Bills, 90 percent of the fair market value of the mortgage loans, and 80 percent of the fair market value of the fixed assets. You may assume the cash reserves at the fed may be liquidated at short notice for 100% of their fair value.

You are asked to estimate the liquidity index using the following formula:

$$I = \sum_{i=1}^{N} w_i \frac{P_i}{P_i'}$$

Notation

- P_i: fire-sale asset prices
- P_i' : fair market prices
- w_i: portfolio weights
- $i \in \{1, 2, \ldots, N\}$ is the asset index

Estimate the liquidity index using the above information.

89. (LRM-126)

A DI with the following balance sheet (in millions) expects a net deposit drain of $15 million.

Assets	Liabilities
• Cash = 10	• Deposits = 68
• Loans = 50	• Loans = 7
• Securities = 15	

(a) Verify that total assets equal total liabilities

(b) Determine the balance sheet after the DI purchases liabilities to offset this expected drain

105. (LRM-129)

Suppose that $f(r)$ is a function that states the cumulative variance explained by the first r principal components. A large dataset X is used with a total number of $K = 1000$ factors.

You are given the table below, which shows results from a principal component analysis (PCA):

r	$f(r)$
1	$\frac{1}{1000}$
2	$\frac{2}{1000}$
3	$\frac{3}{1000}$
4	$\frac{4}{1000}$
5-999	$\frac{r}{1000}$
1000	100%

Provide a recommendation for whether PCA should be used with $r = 3$.

124. (LRM-130)

You are asked to compute the aggregate risk using a bottom-up approach for three risks.

The three risks are Risk A, Risk B, and Risk C.

The correlation matrix between the risks is given by: $\Sigma = \begin{bmatrix} 1 & .5 & .2 \\ .5 & 1 & .3 \\ .2 & .3 & 1 \end{bmatrix}$

Suppose that on a stand-alone basis, each risk is 100.

(a) Compute the aggregated risk using a bottom-up approach. You should calculate the aggregate risk number by using the correlation matrix between the three risks.

You present the results of your analysis from (a) to the board, but they are not familiar with a variance-covariance matrix. The CFO asks if you can look at simpler risk aggregation models that are easier to understand.

(b) Compute the aggregated risk using a simple summation approach

(c) Compute the aggregated risk using a fixed diversification percentage approach, assuming 20% diversification benefit

You present these new results to the board, but they do not like the new results because the aggregated risk numbers are larger. The CFO mentions that the 20% diversification benefit number seems arbitrary. He asks if you can solve for the percentage that would give the same aggregated risk number as (a).

(d) Solve for the fixed diversification x that gives an aggregated risk number equal to the results from (a)

The CFO is happy with your results from (d). He determines that this will be the approach used. To support his decision, he asks that you provide three advantages of using this approach.

(e) State three advantages of the fixed diversification percentage risk aggregation approach

125. (ESWG Report)

For a single scenario in the Academy's interest rate generator, you are given the following projected nominal rates on a real world basis as of a single point in time:

- Long rate = 7.26%
- Sample mid-term 7-year rate = 6.41%
- Short rate = 4.10%

The best fit curve is:

Maturity	Yield
3 months	3.96%
6 months	4.00%
1 year	4.12%
2 years	4.77%
3 years	5.11%
5 years	5.93%
7 years	6.38%
10 years	6.71%
20 years	7.23%
30 years	7.41%

Complete the final yield curve as of this point in time.

132. (LRM-120)

A liability consists of a series of 15 annual payments of 35,000 with the first payment to be made one year from now.

The assets available to immunize this liability are five-year and ten-year zero-coupon bonds.

The annual effective interest rate used to value the assets and the liability is 6.2%. The liability has the same present value and duration as the asset portfolio.

Calculate the amount invested in the five-year zero-coupon bonds.

136. (LRM-120)

You are given the following information about an security where all present values are determined with the same yield:

Time	Cash Flow	Present Value of Cash Flow
1	100.00	96.15
2	100.00	92.46
3	200.00	177.80
4	200.00	170.96
5	300.00	246.58
Total	900.00	783.95

Calculate the Modified duration of the security.

150. (LRM-117: Key Rate Durations)

A GNMA Pass Through portfolio is made up of interest-only and principal-only securities.

You are given the following information:

- The GNMA Pass Through portfolio is priced at $300.
- The interest-only security is priced at $100 and the principal-only security is priced at $200.
- At the first key rate of 1 year, the interest-only security has a key rate duration of 0.05 and the principal-only security has a key rate duration of -0.08.

Calculate the portfolio's key rate duration at the first key rate.

151. (LRM-117: Key Rate Durations)

You have been given the following key rate durations and you know the effective duration is 5.

Term (Years)	**0.25**	**1**	**2**	**3**	**5**	**7**	**10**	**15**	**20**	**25**	**30**	**Total**
Duration (Years)	0.02	0.19	0.35	0.56	0.69	0.74	0.65	0.61	0.34	0.28	?	**?**

Complete the table above and determine at which key rates are you making interest rate bets.

