QFII-104-14 Correlation Pitfalls And Alternatives

Elliptical Distributions

Linear correlation is unproblematic when dealing with **elliptical distributions**

- **Examples**: Multivariate normal distribution, multivariate t-distribution

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    - Outside the elliptical world, there are infinitely many multivariate distributions that could fit
    - Correlation does not tell us anything about the degree of dependence in the tails of the underlying distribution
2. **Given marginal distributions F_1 and F_2 for X_1 and X_2, all linear correlations between -1 and 1 can be attained through a suitable specification of the joint distribution F**
    - In general, the attainable correlations depend on F_1 and F_2, and form a closed interval $[\rho_{min}, \rho_{max}]$ (containing zero) that is a subset of $[-1, 1]$

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    - **Example**: $\ln(X_1)$ and $\ln(X_2)$ do not have the same correlation as X_1 and X_2
6. Correlation is only defined when the variances of the risks are finite
    - Not appropriate for very heavy-tailed risks where variances appear infinite

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1. **Kendall's rank correlation**

$$\rho_S(X_i, X_j) = \rho\left(F_i(X_i), F_j(X_j)\right)$$

Disadvantages:

- Still has deficiencies 1 and 4 listed in previous slide
- Cannot be manipulated as easily as linear correlation, resulting in analytical difficulties when calculating variance of a portfolio's return
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2. **Copulas**

- Represents a way of extracting the dependence structure from the joint distribution
- By simply applying a copula function to prescribed marginal distributions, we can construct a multivariate distribution
- Best way to model dependency between risks! ✓