Statutory Valuation of ILA Contracts, 5th ed.

Source Author: Claire, Lombardi, and Summers (2018)

Chapter 11: Valuation Methodologies

Pages Included on Syllabus: 215–242

Overview of This Reading

This is one of the most critical and foundational chapters on the entire LFV syllabus

Even though we're very much in a statutory reserving context—specifically **SVL Section 5**—the reserve mechanics covered here "ripple" into other areas of the syllabus like the GAAP material

I strongly recommend watching the previous Actuarial Math Review video lesson before working through the reserve formulas in this chapter—doing so will refresh you on basic first principles concepts and allow you to focus more energy on the terminology and methods in this chapter

The first half of this chapter covers net premium reserve methods used for statutory reserving

The second half of the chapter describes various approximations and additional liabilities that companies implement in practice

The detailed study manual is very thorough for this material, but based on my experience as an instructor, it is impossible to learn these reserve methods without working actual numerical examples. The video lesson provides extensive examples for this purpose.

Key topics for the exam include:

1. Common reserve methodologies—be able to compare each and perform calculations for each
    - Net level premium method
    - Modified reserve method
    - Full preliminary term method
    - CRVM
2. Common approximations and other liabilities
    - Mean reserves
    - Mid-terminal reserves
    - Deferred premium asset vs. unearned premium liability
    - Continuous reserves—semi-, fully, and discounted
    - Immediate payment of claims reserves

Terminology and Notation

Valuation methodology – the particular **net premium method** used to determine the policy reserve

The 2 most common net premium methods are the net level premium method (NLP) and Commissioners Reserve Valuation Method (CRVM)

Both center around the basic formula:

Reserve = Present value of benefits − Present value of **net premiums**

Key Dates

1. **Issue date** – start date of policy
2. **Maturity date** – end date of policy
3. **Valuation date** – date on which policy reserve is measured
   - Must be the same as the financial statement (balance sheet) date
   - Policy data may require adjustments if not "as of" the valuation date

Time periods and symbols

1. **Benefit period** = Maturity Date – Issue Date = n
2. **Premium paying period** = $m \leq n$
   - Sometimes the premium-paying period is less than the benefit period (e.g. a 20-pay whole life policy)
3. **Policy year** = number of years from the issue date to a specific valuation date, rounded up to the nearest whole year

   Starts at 1: For example, immediately after policy issue, you're already in the first policy year
4. **Policy duration** = number of years from the issue date to a specific valuation date, rounded down[8]
   - Same example as before: policy duration = 0 until you reach the first anniversary
5. **Issue age** = $[x]$
   - Contrast this with attained age x

[8] In practice, many companies define policy duration the same as policy year, so throw out anything you know from the "real world" and stick with Lombardi's definition for this material

About the SVILAC Textbook Examples

The examples in this chapter are very long. The tables you see in this and other chapters are generated from spreadsheet programs that ship with the 5th edition of the book.

The current syllabus notes that you are not required to use or know the spreadsheet content for the exam, but it encourages you to use the spreadsheets for extra clarification. A word of caution though: you will NOT have Excel in the exam room, and I think the very best use of your finite study time is to learn how to work these reserve calculations using the tools you'll have on exam date: pen/pencil and SOA-approved calculator. So be careful about losing hours in Excel, especially on things that are not explicitly covered in the textbook itself!

The video lessons for this chapter and other SVILAC chapters are based on shorter versions of the book examples and can be done using a hand-held calculator. So they are more appropriate for an exam setting and more useful for learning the material using the tools you have available.

Reserve Notation

The actuarial notation in SVILAC is very technical and can be distracting at times because it's generalized for limited-pay products and term products. We're going to use simpler (but generally accepted) versions in the detailed study manual and video lessons to avoid distracting you from the key concepts.

Primary terms using the SVILAC book notation:

$$
\begin{aligned}
{}^{m}_{t}V_{[x]:\overline{n}|} &= \text{NLP reserve at duration } t \text{ for an } m\text{-pay product} \\
AB_{[x]+t:\overline{n-t}|} &= \text{PV future benefits at duration } t \\
\ddot{a}_{[x]+t:\overline{m-t}|} &= \text{PV of annuity-due at duration } t \\
{}_{m}PB_{[x]:\overline{n}|} &= \text{Net level benefit premium for an } m\text{-pay product} \\
{}_{m}PE_{[x]:\overline{n}|} &= \text{Net level expense premium for an } m\text{-pay product}
\end{aligned}
$$

In the online seminar, we will generally use simpler notation:

$$
\begin{aligned}
x &= \text{issue age (no brackets for simplicity)} \\
x+t &= \text{attained age at duration } t \\
{}_{t}V_{x} &= \text{Reserve at time } t \text{ for issue age } x \\
\text{PVFB}_t &= \text{PVFB at time } t \text{ (} = A_x \text{ for a level \$1 DB)} \\
\ddot{a}_{x+t} &= \text{PV of an annuity-due at time } t \\
PB_t &= \text{Net benefit premium at duration } t \\
PE_t &= \text{Net expense premium at duration } t \\
NP_t &= \text{Total net premium } (PB_t + PE_t) \\
\text{PVNP}_t &= \text{PV of future net premiums at duration } t \\
\text{PVGP}_t &= \text{PV of future gross premiums at duration } t
\end{aligned}
$$

On the exam, using simpler notation is fine. Please don't lose precious time on the exam trying to reproduce all of the superscripts and subscripts in the SVILAC book! In the video lesson, we give more tips.

Common Methodologies

There are 4 methodologies covered in the next few sections, but they are not mutually exclusive

High level summary:

1. Net level premium (NLP) method
   - Reserve net premium is a level % of GPs
   - No explicit recognition of expenses (of any kind) or lapses
   - Typically largest reserve of the 3 methods
2. Modified reserve method
   - Any NLP method that also includes an expense allowance (EA), which lowers reserves
   - FPT and CRVM are modified methods that are different only in the way they define the EA

These methods are discussed in more detail below, and the video lesson has a numerical example for each

Net Level Premium Method (NLP)

For a basic WL policy with \$1 DB and level premiums payable for life, the NLP reserve is:

$$ {}_tV_x^{NLP} = A_{x+t} - \frac{A_x}{\ddot{a}_x}\ddot{a}_{x+t} $$

More general case when DBs and/or GPs vary by year:

$$ \begin{aligned} {}_tV_x^{NLP} &= \text{PVFB}_t - \text{PVNP}_t \\ &= \text{PVFB}_t - NP_0 \cdot \ddot{a}_{x+t} \\ r_t^{GP} &= \text{gross premium ratio} = \frac{GP_t}{GP_0} \\ NP_0 &= \left(\frac{\text{PVFB}_0}{\ddot{a}_x}\right) = \text{NP for first policy year} \\ \ddot{a}_x &= 1 + v \cdot {}_1p_x \cdot r_1^{GP} + v^2 \cdot {}_2p_x \cdot r_2^{GP} + \cdots \\ \ddot{a}_{x+t} &= r_t^{GP} + v \cdot {}_1p_{x+t} \cdot r_{t+1}^{GP} + v^2 \cdot {}_2p_{x+t} \cdot r_{t+2}^{GP} + \cdots \end{aligned} $$

Important things to understand from these formulas:

- r_t^{GP} is the ratio of the GP at duration t to the GP <u>at issue</u> (even for $\ddot{a}_{x+t}$!)

- The net premium is a constant % of GPs

$$\frac{NP_t}{GP_t} = \frac{NP_0 \cdot r_t^{GP}}{GP_0 \cdot r_t^{GP}} = \frac{NP_0}{GP_0}$$

- The formula for $\ddot{a}_{x+t}$ reflects the GP pattern
  - Therefore you always multiply NP_0, which is determined at issue for the NLP method
  - BUT the net premium does vary in dollars: $NP_t = NP_0 \cdot r_t^{GP}$, which is effectively the first term in the PVNP$_t$ calculation

All of the other methods can also have varying GPs, so remember to use this definition of an annuity-due in general for any method. Even for methods like FPT that determine a NP at $x+1$, you still "anchor" r^{GP} at issue. All of this is demonstrated using examples in the video lesson.

Modified Reserves

A net level premium method that includes an **expense allowance (EA)**

- EA = a formula that reflects that most life insurance contracts require large first year expenses (acquisition costs, etc.)
- Under SVL, the EA does NOT represent actual expenses (*very different from the way U.S. GAAP recognizes first year acquisition expenses using a DAC asset—we will cover that in the GAAP material later*)

The EA is amortized over the premium paying period and has the effect of lowering the stat reserve

$$\text{Modified Reserve}_t = \text{NLP Reserve} - \text{Unamortized EA} \tag{1}$$

$${}_tV_x^{Mod} = {}_tV_x^{NLP} - {}_tVE_x \tag{2}$$

$$= \text{PVFB}_t - [\text{PVPB}_t + \text{PVPE}_t] \tag{3}$$

$${}_tVE_x = PE_0 \times \ddot{a}_{x+t} \tag{4}$$

$$PE_0 = \frac{EA_x}{\ddot{a}_x} \tag{5}$$

$$PB_0 = \frac{\text{PVFB}_0}{\ddot{a}_x} \tag{6}$$

$$NP_t = PB_t + PE_t \tag{7}$$

Describing the above equations in words:

Equation 5: Determine a net expense premium (PE) for the EA using the same mechanics we used to calculate a NP for future benefits ⇒ PV of PEs must equal the EA at issue

- Think of the PE as an additional "slice" of the GP that you're claiming for the EA

  - PE is an additional "allocation" of the GP over and above the portion allocated for the benefit net premium, PB
  - The annuity-due is exactly the same as the NLP Method (reflects GP pattern, etc.)

  Equation 4: The unamortized EA is always equal to the PV of future PEs remaining

  - The unamortized EA is essentially an "asset" that partially offsets the NLP reserve (equation 2)

- **Equation 7:** Since you're effectively using a higher net reserve premium, the reserve must be lower
  - The term in brackets in equation 3 is the PV of total NPs
  - If GPs vary, multiply PB_0 and PE_0 by r_t^{GP}

Remember: The EA is simply a formulaic mechanism that results in lower NLP reserves

- This section simply introduces the concept of having an EA > 0
- The actual EA will vary depending on the type of modified reserve method (FPT vs. CRVM)
- EA ≠ actual first year expenses (*worth repeating!*)

Full Preliminary Term Reserve (FPT)

FPT is a modified reserve with an expense allowance defined as

$$\begin{aligned} EA_x^{FPT} &= \left(\frac{\text{PVFB}_1}{\ddot{a}_{x+1}}\right) - c_x \\ &= NP_1 - c_x \\ c_x &= v \cdot q_x \cdot \text{DB} = \text{first-year cost of insurance} \end{aligned}$$

Conceptually, the NP for $t \geq 1$ is calculated as though the policy was sold at age $x + 1$

$$NP_t = \begin{cases} c_x & \text{for } t = 0 \\ \dfrac{\text{PVFB}_1}{\ddot{a}_{x+1}} \cdot r_t^{GP} & \text{for } t \geq 1 \end{cases}$$

At the end of the first year, the FPT reserve is zero because $NP_1 = NP_0$ for the same policy issued to $x + 1$ under the NLP Method

$$\begin{aligned} {}_1V_x^{FPT} &= \text{PVFB}_1 - NP_1 \cdot \ddot{a}_{x+1} \\ &= \text{PVFB}_1 - \left(\frac{\text{PVFB}_1}{\ddot{a}_{x+1}}\right) \cdot \ddot{a}_{x+1} \\ &= 0 \end{aligned}$$

Alternatively, using a retrospective approach:

$$\begin{aligned}
{}_1V_x^{FPT} &= \frac{\left({}_0V_x^{FPT} + NP_0\right)(1+i) - q_x}{p_x} \\
&= \frac{(0 + c_x)(1+i) - q_x}{p_x} \\
&= \frac{(0 + vq_x)(1+i) - q_x}{p_x} \\
&= \frac{q_x - q_x}{p_x} \\
&= 0
\end{aligned}$$

In other words, the first year's NP is exactly enough to pay the mortality cost for the first year, so the reserve is completely exhausted by EOY 1

More general notation:

- α = the first year's net premium = c_x under FPT
- β = net premiums for all other years

Commissioners Reserve Valuation Method (CRVM)

CRVM is just like FPT, only it limits the EA that can be used

AG 27 defines the CRVM EA as the smaller of:

1. EA under FPT for the contract
2. EA under FPT assuming 20-pay WL contract

If the 20-pay EA limitation applies, the CRVM reserve will be greater than zero at EOY 1

- This also means the first year NP > c_x in these situations

Additional rules for the 20-pay EA limitation:

1. If the actual contract has non-level DBs, calculate the 20-pay EA using a level DB = average actual DB for policy years 2–10
   - This would affect NP_1, which you need to calculate $EA = NP_1 - c_x$
   - Example: suppose contract's DB increases from 100,000 to 150,000 at $t = 5$:

$$\text{Level DB to Assume} = \frac{4(100{,}000) + 5(150{,}000)}{9} = 127{,}778$$

2. If the EA is negative, set equal to zero (per AG 21)
   - May occur if GPs increase steeply resulting in $NP_1 < c_x$

- I.e. the more steeply GPs rise, the lower early NPs will be because they will "catch up" in later years as r_t^{GP} rises

FAQ: Does the same EA limitation apply to term insurance? *Yes! It applies to all life insurance subject to SVL. However, term policies often cover short benefit periods (e.g. 10-year term). This will often result in a lower NP_1 than a WL policy. As a result, many term policies will naturally have an EA < than a 20-pay WL policy. You still have to do the test to be sure.*

CRVM key takeaways:

- Smallest reserves allowable by SVL
- Usually same as FPT unless contract can be paid up more quickly than 20 years

$$_tV_x^{CRVM} \geq {}_tV_x^{FPT}$$

- If premium payment period < 20 years, CRVM reserves will be higher than FPT reserves because the CRVM EA is capped

Other Methods

One common alternative to holding only CRVM reserves is to grade from CRVM to net level reserves over a period of time

- Allows a company to offer products with 20th-year CSVs > minimum required CSVs but not greater than the reserve held (avoids having to hold an excess CSV liability)

Common Approximations and Other Liabilities

The primary theme of the remaining material in this chapter is how to address various practical issues that arise in the real world. This usually involves either adjusting the reserve based on previously discussed methods and/or adding additional liabilities.

Stat valuations include many policies at once, so the valuation date often does not occur precisely on a policy's anniversary

Terminal reserve = reserve at end of policy year = $_tV_x$

This is what we've focused on so far

Initial reserve = reserve at beginning of policy year = prior year's terminal reserve + net premium

$$_{t-1}V_x + NP_t$$

Mean and mid-terminal reserves are equivalent ways of determining a reserve for a policy when the valuation date is between anniversaries

Mean Reserves

Interpolated mean reserve = weighted average of initial and terminal reserve

$$(1-h)\,({}_{t-1}V_x + NP_t) + h\,({}_tV_x)$$

where h = fraction of the policy year between previous anniversary and valuation date

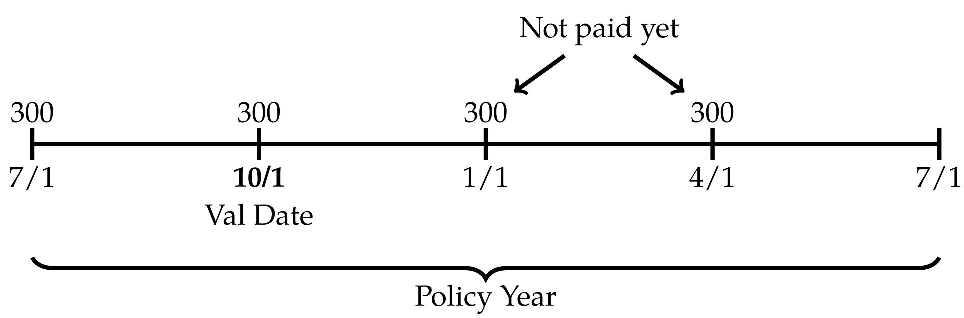
- Example: If the policy anniversary is July 1 and the valuation date is October 1, $h = 3/12$

If the policyholder pays premiums more frequently than annually, subtract a **deferred premium asset (DPA)** from the interpolated mean reserve above

The DPA reflects that not all of the reserve net premium has been collected

DPA = sum of modal premiums due between valuation date and next anniversary

- If the annual NP is 1200 and the policyholder pays quarterly, modal net premium = 300
- Using the dates in the previous example: DPA at October 1 = 600 = the 2 modal premiums that will be paid on January 1 and April 1



- Note that the DPA would also be 600 if the valuation date was 11/1 or 12/1

As an approximation for a group of policies, most companies assume any given valuation date falls in the middle of each policy year on average ($h = 0.5$), which allows calculation of mean reserve factors

Mean reserve = interpolated mean reserve with $h = 0.5$

$$\text{MeanV} = \frac{({}_{t-1}V_x + NP_t) + ({}_tV_x)}{2}$$

Commonly, when exam questions ask you to calculate a "mean reserve," the above formula is what they have in mind.

Mid-Terminal Reserves

Interpolated terminal reserve = weighted average of previous and next terminal reserve

$$(1-h)\,({}_{t-1}V_x) + h\,({}_tV_x)$$

Since the interpolated terminal reserve doesn't reflect the net premium at all, must add an **unearned premium liability (UPL)**

UPL = portion of collected premium that hasn't been earned in the current policy year

$$\text{UPL} = \frac{\text{\# months until next premium}}{\text{\# months between premium payments}} \times \text{Modal NP}$$

Extending the previous example with a 300 modal net premium. . .

- If val date = Oct 1: UPL = $^3/_3 \times 300 = 300$
- If val date = Nov 1: UPL = $^2/_3 \times 300 = 200$
- If val date = Dec 1: UPL = $^1/_3 \times 300 = 100$

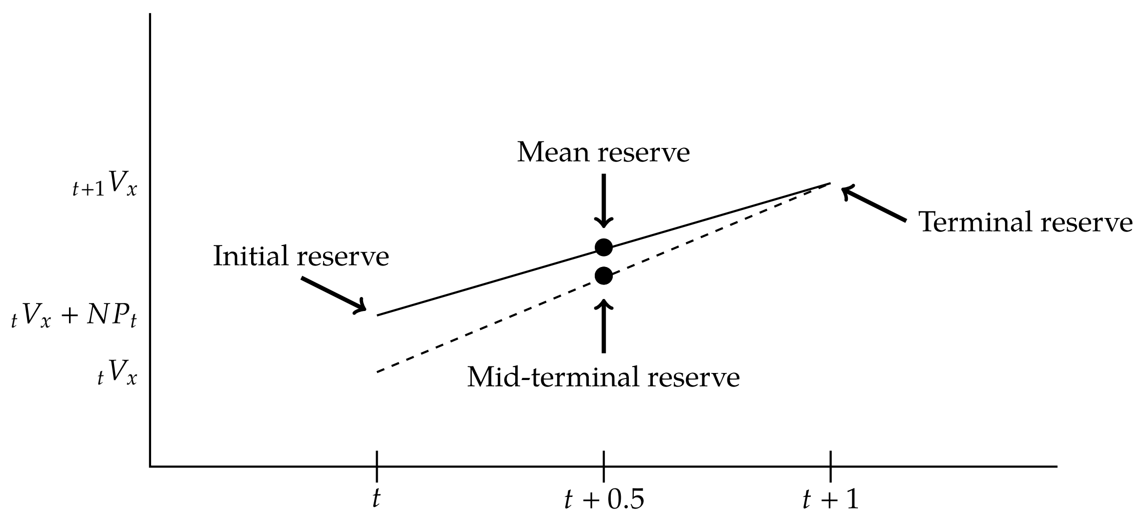
Mid-terminal reserve = interpolated reserve at mid-year ($h = 1/2$)

$$\text{MidV} = \frac{{}_{t-1}V_x + {}_tV_x}{2}$$

For mid-terminal reserves, the UPL is usually approximated as $^1/_2 \times$ Modal NP

Similar to my comments on mean reserves, when exam problems ask for "mid-terminal reserves," the above approximation is usually what they have in mind.

Comparing Mean and Mid-Terminal Reserves



In all cases: Mean Reserve − DPA = Mid-Terminal Reserve + UPL

- For annual mode policies, the correct reserve would travel along the mean reserve line (DPA = 0)

$$\text{Mean Reserve} = \text{Mid-Terminal Reserve} + \text{UPL}$$

The UPL would exactly "correct" the mid-terminal reserve, bringing it up to the mean reserve line

For example, for an annual mode policy at mid-year ($h = 1/2$):

$$\text{Mean Reserve} = \text{Mid-Terminal Reserve} + \text{UPL}$$
$$\frac{1}{2}\left({}_{t-1}V_x + NP_t\right) + \frac{1}{2}\left({}_tV_x\right) = \frac{{}_{t-1}V_x + {}_tV_x}{2} + \frac{6}{12}NP_t$$

- For non-annual modes, the correct reserve would fall somewhere between the mean and mid-terminal lines

Immediate Payment of Claims Reserve (IPCR)

Required by AG 32 for any method that assumes curtate EOY DBs (which all the methods have so far)

IPCR reflects that DBs are actually paid at the time of death rather than at the EOY

If the contract provides for payment of DB immediately upon receipt of due proof of death without interest from the date of death:

$$\text{IPCR}_t = \frac{i}{3} \times \text{PVFB}_{x+t}$$

If interest is paid on the DB:

$$\text{IPCR}_t = \frac{i}{2} \times \text{PVFB}_{x+t}$$

AG 32 refers to PVFB_{x+t} as the "death portion of the reserve" (i.e. the part of the reserve for curtate DBs)

$$\text{PVFB}_{x+t} = \text{DB} \times A_{x+t}$$

Continuous Reserves

All of the preceding formulas were curtate—they assumed that premiums were paid BOY and death benefits paid EOY

Semi-continuous reserves assume death benefits are payable at moment of death and NPs are payable annually at BOY

$$
\begin{aligned}
{}_tV_x &= \overline{A}_{x+t} - NP_t \cdot \ddot{a}_{x+t} \\
NP_0 &= \frac{\overline{A}_x}{\ddot{a}_x} \\
\overline{A}_x &= \frac{i}{\delta} A_x = \text{APV of continuous DBs} \\
\frac{i}{\delta} &= \text{“continuous reserve adjustment”} \\
\delta &= \ln(1+i) = \text{force of interest}
\end{aligned}
$$

Fully continuous reserves assume both continuously payable death benefits and premiums

- Only difference with semi-continuous: use a continuous annuity factor, $\overline{a}_x$

$$\text{Fully Continuous NP} = \frac{\overline{A}_x}{\overline{a}_x}$$

- While the assumption that DBs are payable continuously is realistic, assuming that premiums are payable continuously "does not necessarily have a basis in fact"
- In practice, reserve factors are mid-terminals with UPL

Discounted continuous reserves = semi-continuous reserves with a refund of unearned portion of premium at death

- In widespread use (more practical than fully continuous)
- Uses a net premium equal to:

$$\text{Fully Continuous NP} \cdot \overline{a}_{\overline{1|}}$$

  where $\overline{a}_{\overline{1|}}$ = 1-year continuous annuity certain = $\frac{1-v}{\delta}$

- The only difference with fully continuous is that discounted continuous discounts the premium with one year of interest
- Discounted continuous terminal reserves = fully continuous terminal reserves
- The premium adjustment allows mean reserves to be calculated
- DPA is calculated if mode is not annual